Probing Primordial Non-Gaussianity with Large-Scale Structure

Roman Scoccimarro (NYU)

- K.C. Chan, M. Manera, L. Hui and R.S. (in preparation)

- + A. Berlind and C. McBride

Primordial Non-Gaussianity from Inflation

Gaussianity is a consequence of:

i) inflaton a single scalar field
ii) slowly rolling
iii) in vacuum state
iv) with canonical kinetic terms

if we relax i) we have for the Bardeen potential,

$$\Phi = \phi + f_{\rm NL}\phi^2$$

which implies for it a bispectrum,

$$B = 2 f_{\rm NL} P_1 P_2 + {\rm cyc.} \qquad -10 < f_{\rm NL}^{\rm local} < 74$$

- For biased tracers (galaxies, halos), this model leads to a scale-dependent bias at large scales (Dalal et al 2008),

$$b_1(k) = b_{10} + \Delta b_1(k, f_{\rm NL})$$

with b~1/k^2 at low-k. Thus the power spectrum of galaxies is sensitive to fnl!!

Generic Predictions in Peak-Background Split

We are interested in establishing as rigorously as possible the validity of the local PNG bias formula

$$\Delta b_1(k, f_{\rm NL}) = \frac{2 f_{\rm NL}}{M(k)} (b_{10} - 1)\delta_c$$

and generalizing it to arbitrary (non-local) PNG. Some issues in derivations,

- proper treatment of filter and transfer function effects

- dependence on primordial bispectrum (cannot be just a number)

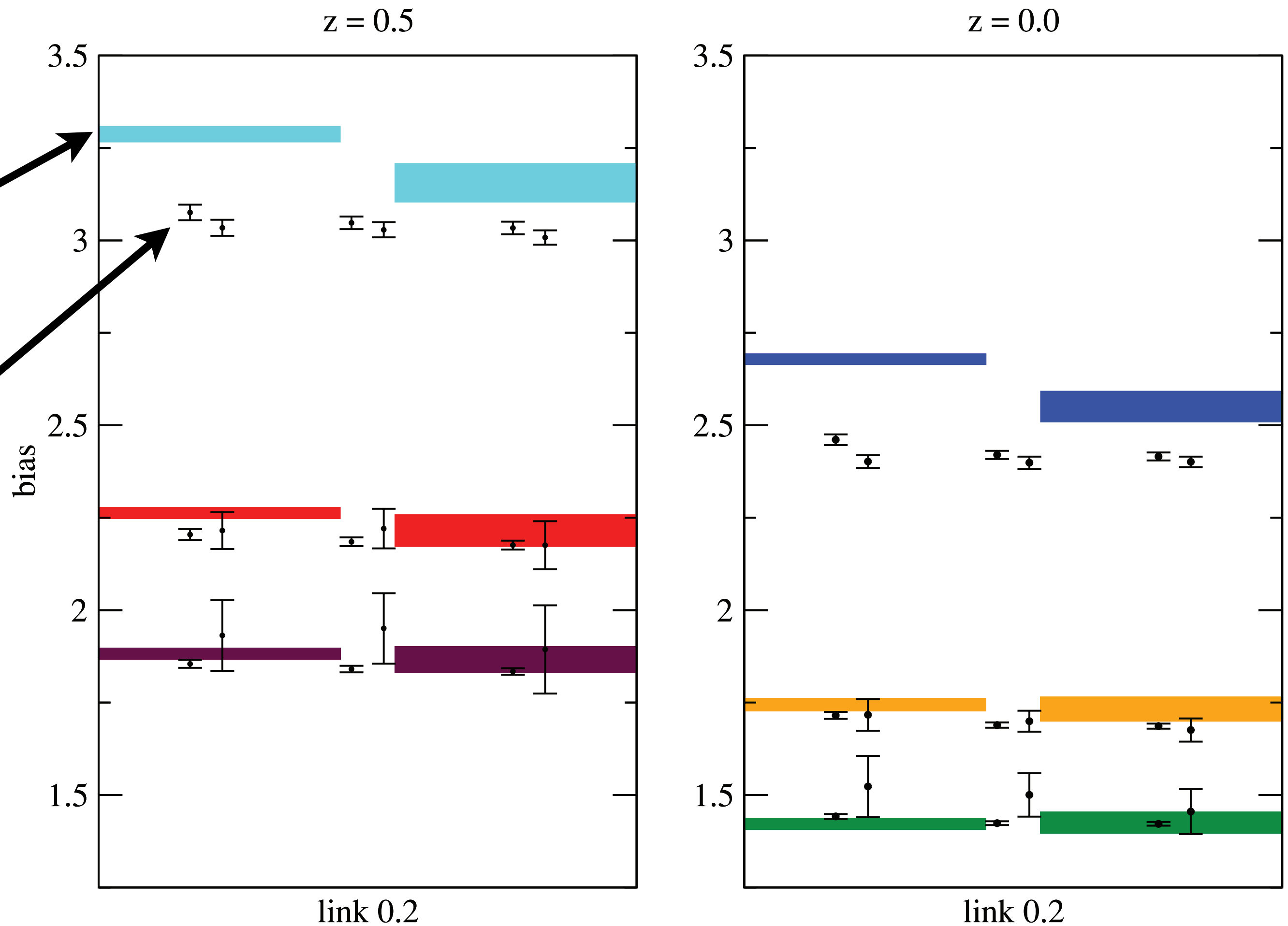
- peaks in phi vs peaks in delta approximations

$$\nabla\phi^2 = 2\phi\nabla^2\phi + 2\nabla\phi\cdot\nabla\phi \approx 2\phi\nabla^2\phi ?$$

simulations suggest a somewhat smaller amplitude (depending on halo def) we know that “PBS” (in std implementation) is not that accurate even in Gaussian case:

measured bias from cross-spectrum

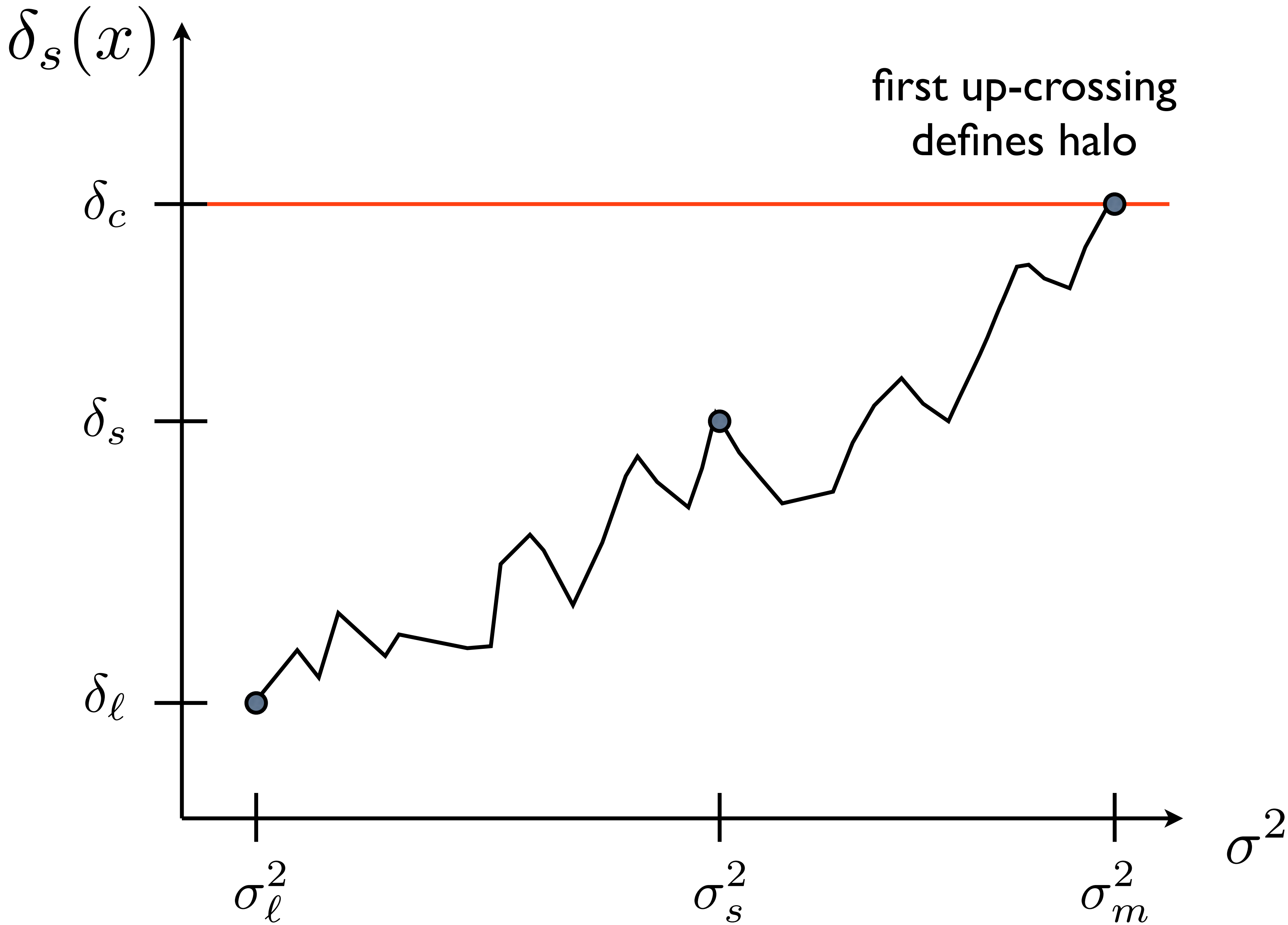
PBS from measured mass function



Manera, Sheth, R.S. (2010)

Figure 11. Comparison of large-scale bias estimates for the same halo mass bins as in previous figures when $l_{\rm link} = 0.2$. Thick bars show the measured $P_{\rm hm}/P_{\rm mm}$ (left-hand panel) and $\sqrt{\xi_{\rm hh}/\xi_{\rm mm}}$ (right-hand panel), and symbols with error bars show the linear bias parameter b_1 predicted from the peak-background split.

Insight into mass function and bias is provided by the excursion-set formalism of halo formation (Bond et al 1991).



A full calculation of the PBS change in bias due to arbitrary PNG bispectrum gives,

$$\Delta b(k) = \frac{\partial_{\sigma^2}\left[I_B(k)\mathcal{F}_0\right]}{M(k)\,\mathcal{F}_0}$$

$$I_B(k,R) \equiv \frac{1}{P_\phi(k)} \int B_{\delta_R \delta_R \phi}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}, -\boldsymbol{k})\, d^3 q$$

Note that, unlike the GW86 formula, what matters is the *cross* bispectrum. For local PNG, expanding in powers of k small (with higher-order corrections coming from filter, transfer function, grad-phi terms, etc

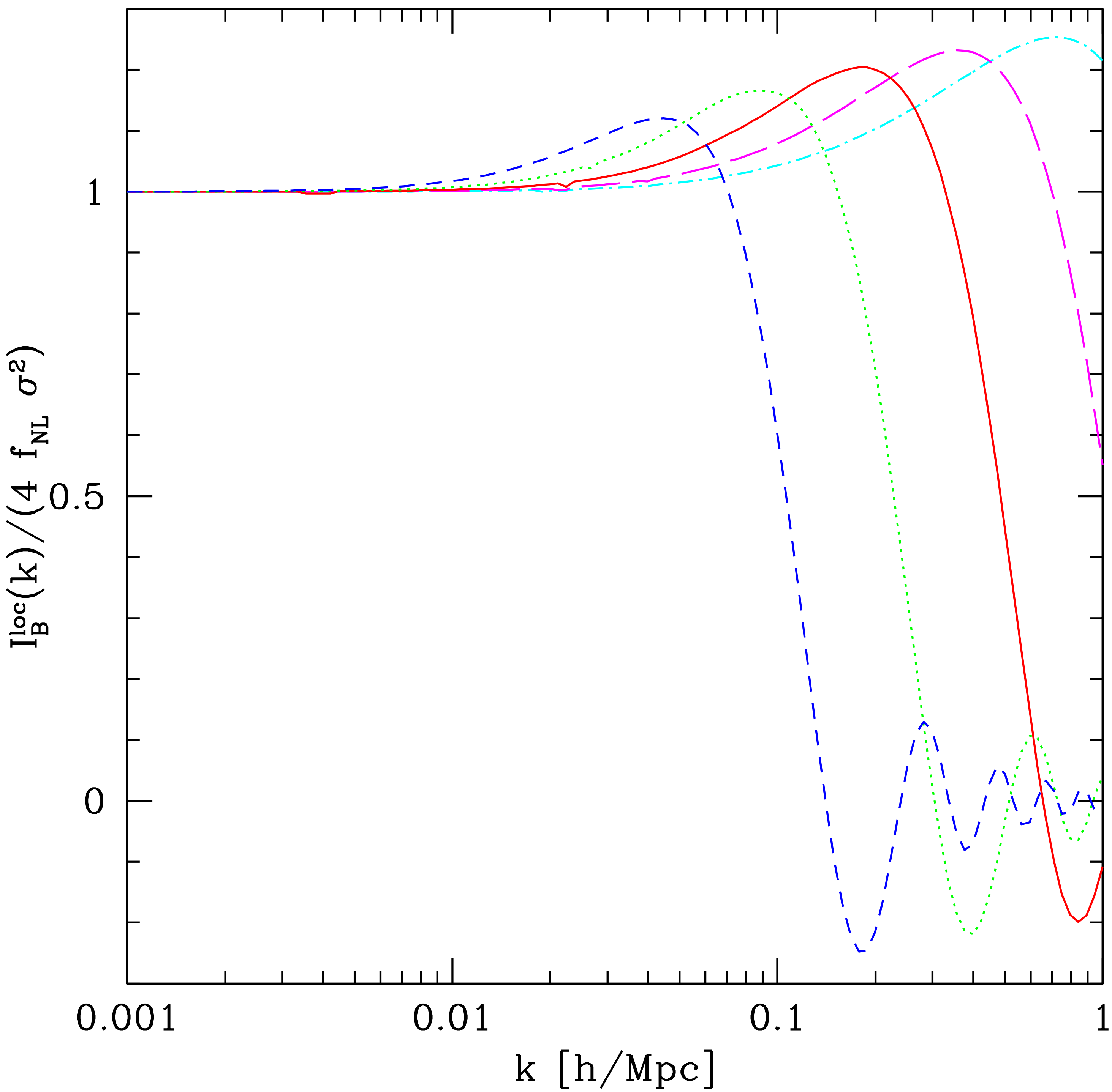
$$I_B(k=0,R) \approx 4 f_{\rm NL}\ {\sigma_R}^2(m) + \mathcal{O}(k^2)$$

which gives

non-markovian ↓

$$\Delta b(k) = \frac{4 f_{\rm NL}}{M(k)}\ \partial_{\ln \sigma^2} \ln(\sigma^2 \mathcal{F}_0) < \frac{2 f_{\rm NL}}{M(k)} \delta_c \frac{(\partial \mathcal{F}/\partial \delta_\ell)_0}{\mathcal{F}_0} = \frac{2 f_{\rm NL}}{M(k)} \delta_c (b_1 - 1)$$

the precise relationship has to be obtained from the first-crossing prob F0.



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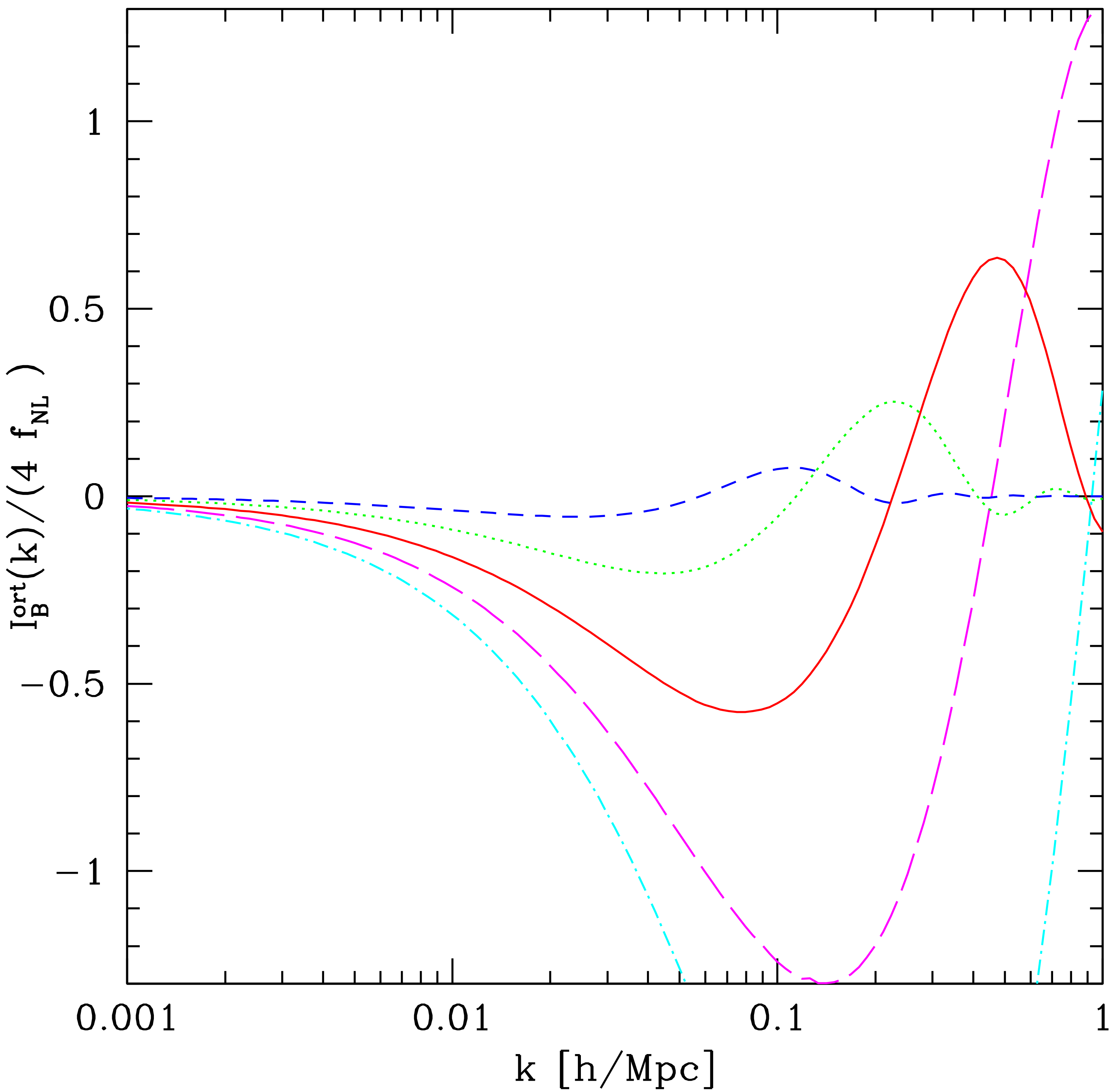

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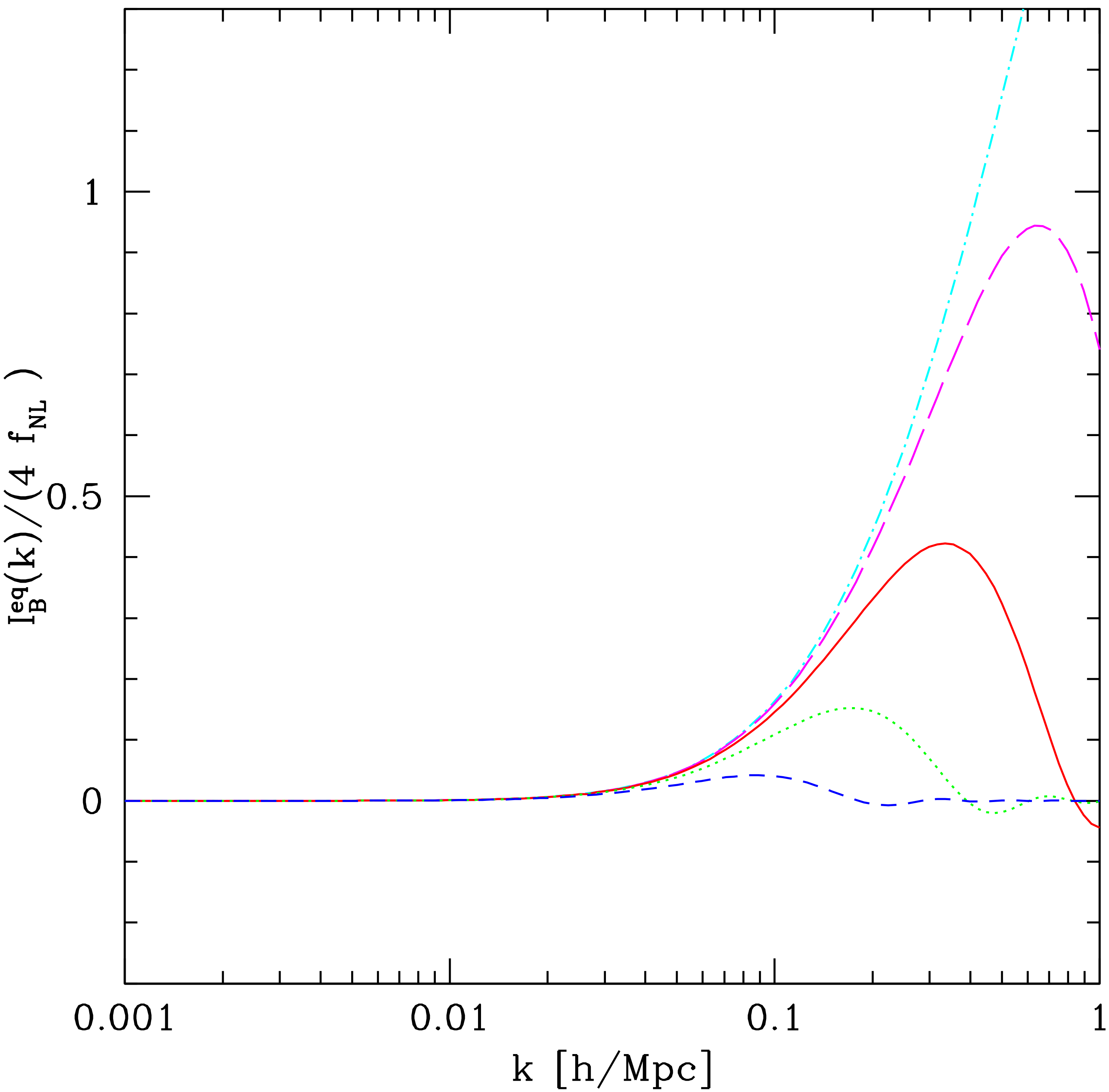
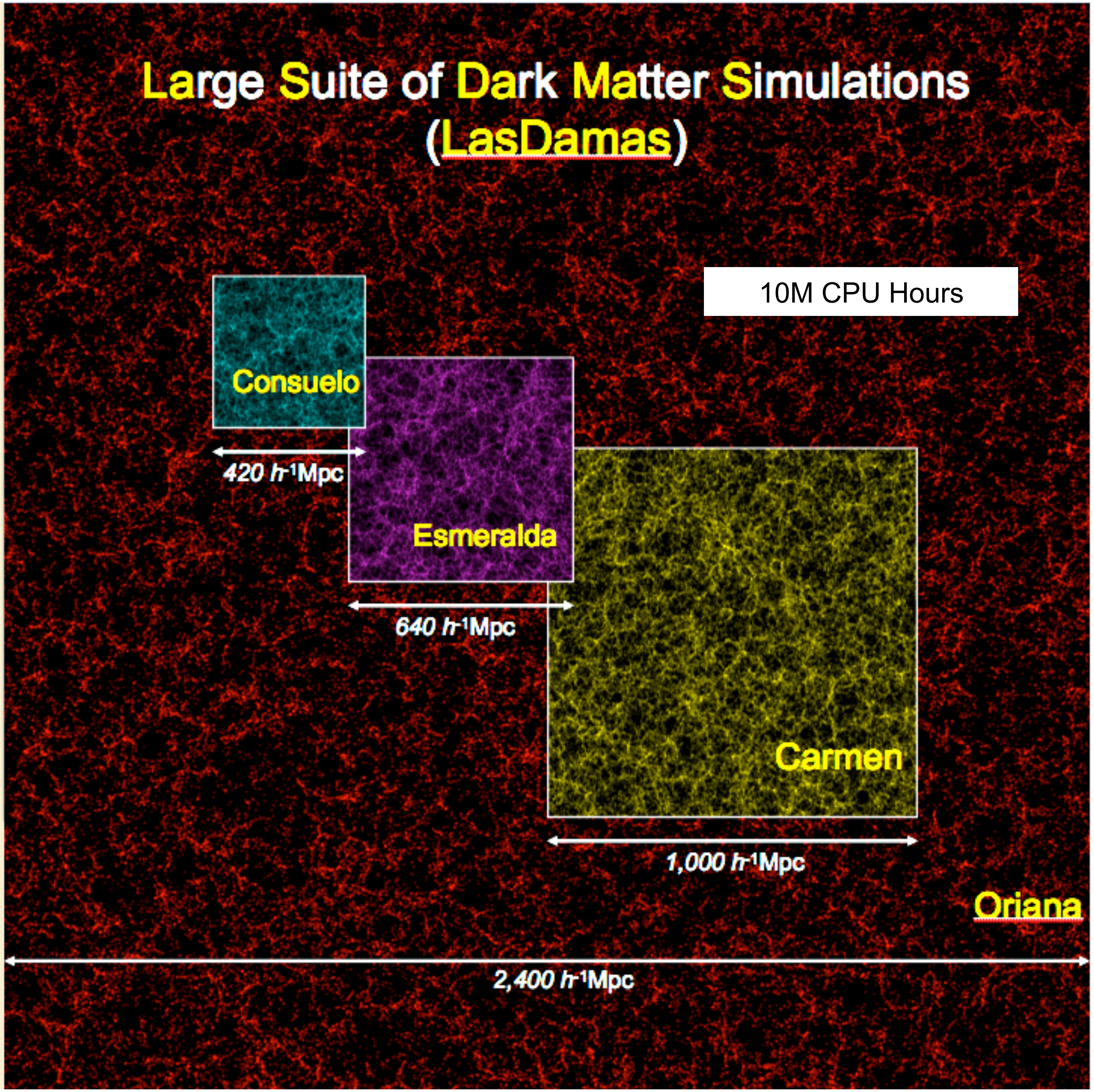


1
0.5
0
0.001
0.01
0.1
1
k [h/Mpc]
$I_B^{eq}(k)/(4 f_{NL})$



with
A. Berlind,
C. McBride,
M. Manera,
J. Gardner,
M. Busha,
R. Wechsler,
F. van den Bosch

LasDamas Simulations

Name	Sample	Lbox	Npar	mpar	Nrealiz
Oriana (G)	LRG +Main -22	2400	1280^3	4.57E+11	42
Oriana fnl_local=+100	LRG +Main -22	2400	1280^3	4.57E+11	12
Oriana fnl_equi=-400	LRG +Main -22	2400	1280^3	4.57E+11	12
Oriana fnl_orto=-400	LRG +Main -22	2400	1280^3	4.57E+11	12
Carmen	Main -21	1000	1120^3	4.98E+10	42
Esmeralda	Main -20	640	1250^3	9.31E+09	50
Consuelo	Main -19-18	420	1400^3	1.87E+09	50

Nmocks=4 x Nrealiz, 2LPT ICs, Gaussian Mocks available at http://lss.phy.vanderbilt.edu/lasdamas/

Large-Scale Bias in non-local PNG: Simulations

- In single-field inflationary models, we are instead interested in models that correspond to non-local PNG (due to non-canonical kinetic terms). For example, the equilateral model has a Bardeen potential bispectrum,

$$(6f_{\rm NL})^{-1}B_{\rm equil} = -P_1P_2 - 2(P_1P_2P_3)^{2/3} + P_1^{1/3}P_2^{2/3}P_3$$

$$-214 < f_{\rm NL}^{\rm equil} < 266$$

(permutations are understood), whereas the orthogonal model template reads

$$(6f_{\rm NL})^{-1}B_{\rm ortho} = -3P_1P_2 - 8(P_1P_2P_3)^{2/3} + 3P_1^{1/3}P_2^{2/3}P_3$$

$$-410 < f_{\rm NL}^{\rm ortho} < 6$$

We are interested in generating such bispectra from quadratic (non-local) models, i.e.

$$\Phi = \phi + f_{\rm NL}\ K[\phi,\phi]$$

where K is the appropriate non-local quadratic kernel that generates the desired bispectrum. For simplicity, here we assume scale-invariance.

- Introduce some handy non-local operators

$$\partial\phi \equiv \sqrt{-\nabla^2}\phi(\mathbf{x}) \equiv \int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\; k\,\phi(\mathbf{k})\,d^3k$$

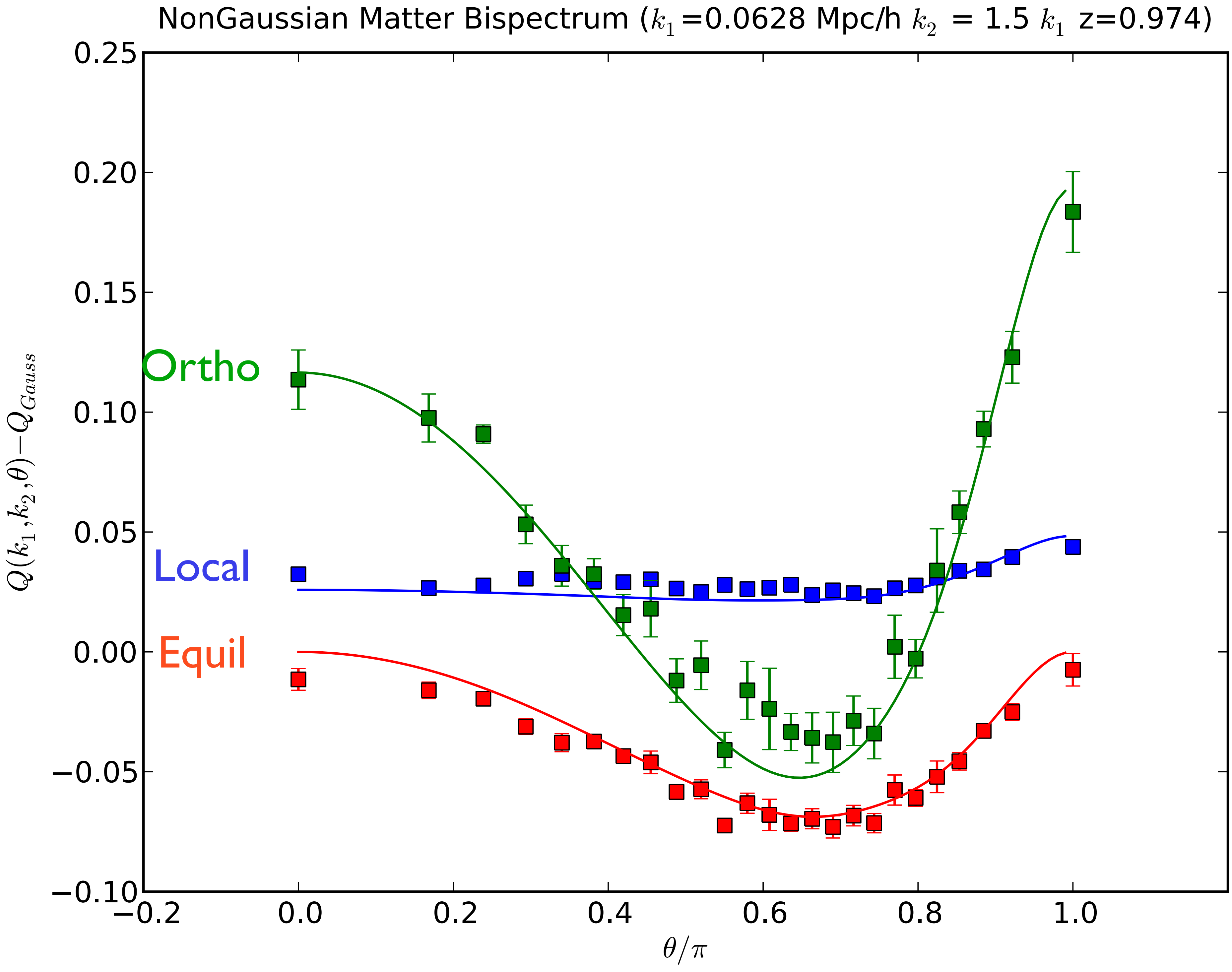
$$\nabla^{-2}A(\mathbf{x}) \equiv -\int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\left(\frac{1}{k^2}\right)A(\mathbf{k})\,d^3k$$

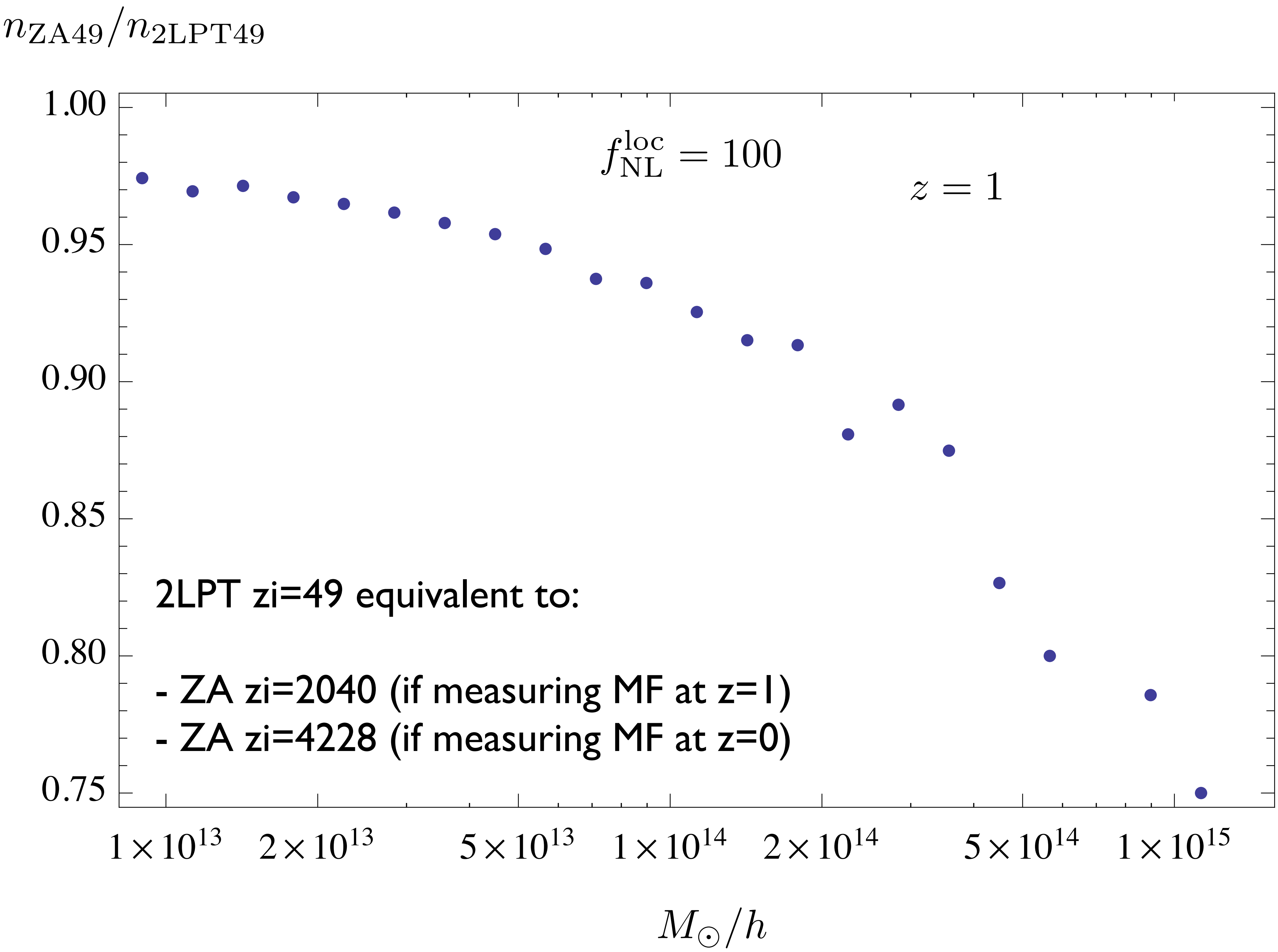
$$\partial^{-1}A \equiv \sqrt{-\nabla^{-2}}A \equiv \int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\left(\frac{1}{k}\right)A(\mathbf{k})\,d^3k$$

Then the EQ and ORT bispectra templates can be generated by,

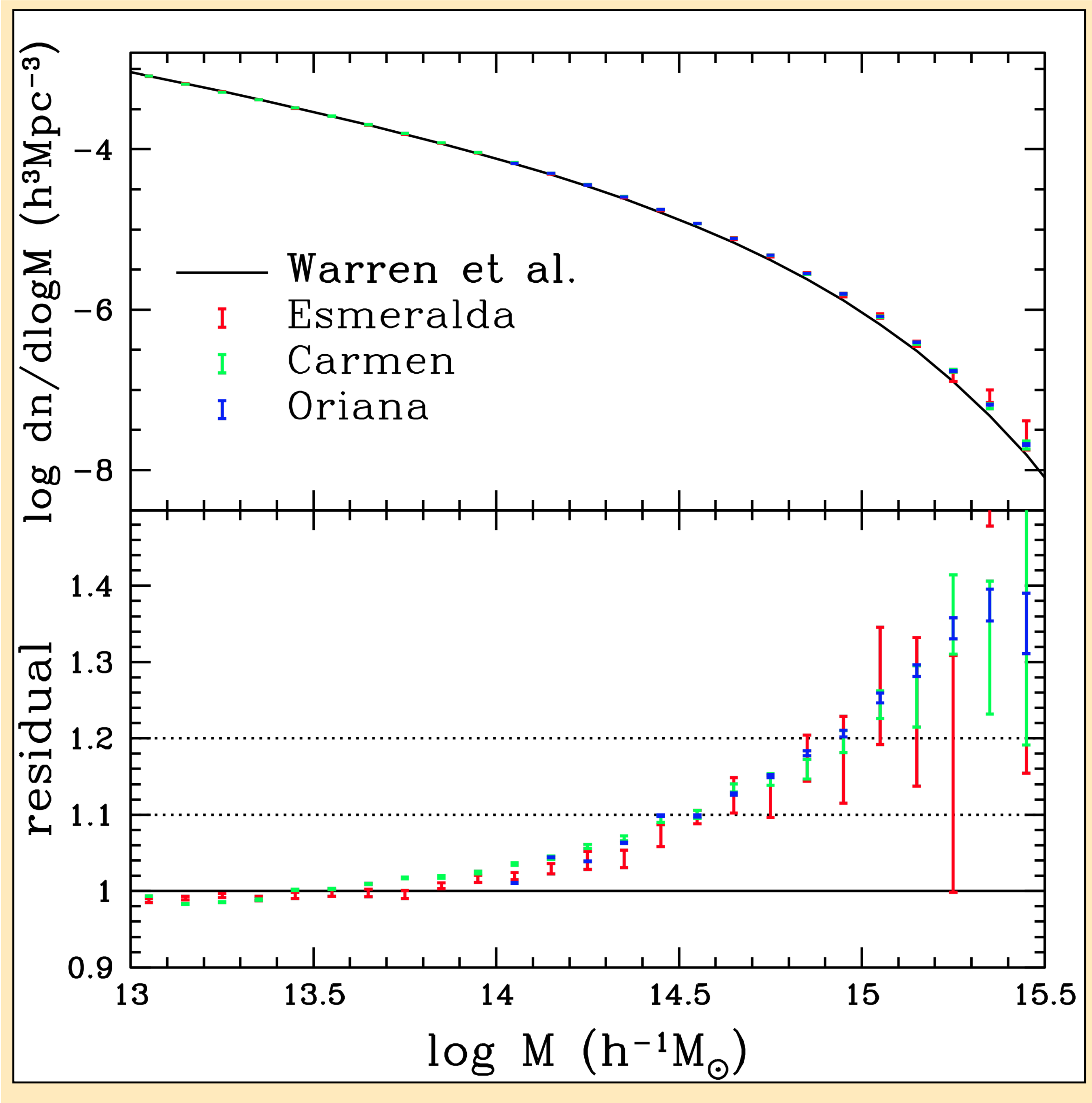
$$K[\phi,\phi] = a\phi^2 + b\,\partial^{-1}(\phi\,\partial\phi) + c\,\nabla^{-2}(\phi\;\nabla^2\phi) + d\,\nabla^{-2}(\partial\phi)^2 + e\,\nabla^{-2}\partial^{-1}(\phi\nabla^2\partial\phi) + f\,\nabla^{-2}\partial^{-1}(\nabla^2\phi\;\partial\phi)$$

regularity constraints (one-loop corrections to the power spectrum must preserve scale-invariance in the IR) restrict the free parameters that leave the bispectrum invariant. Note these kernels have correct exchange symmetry.

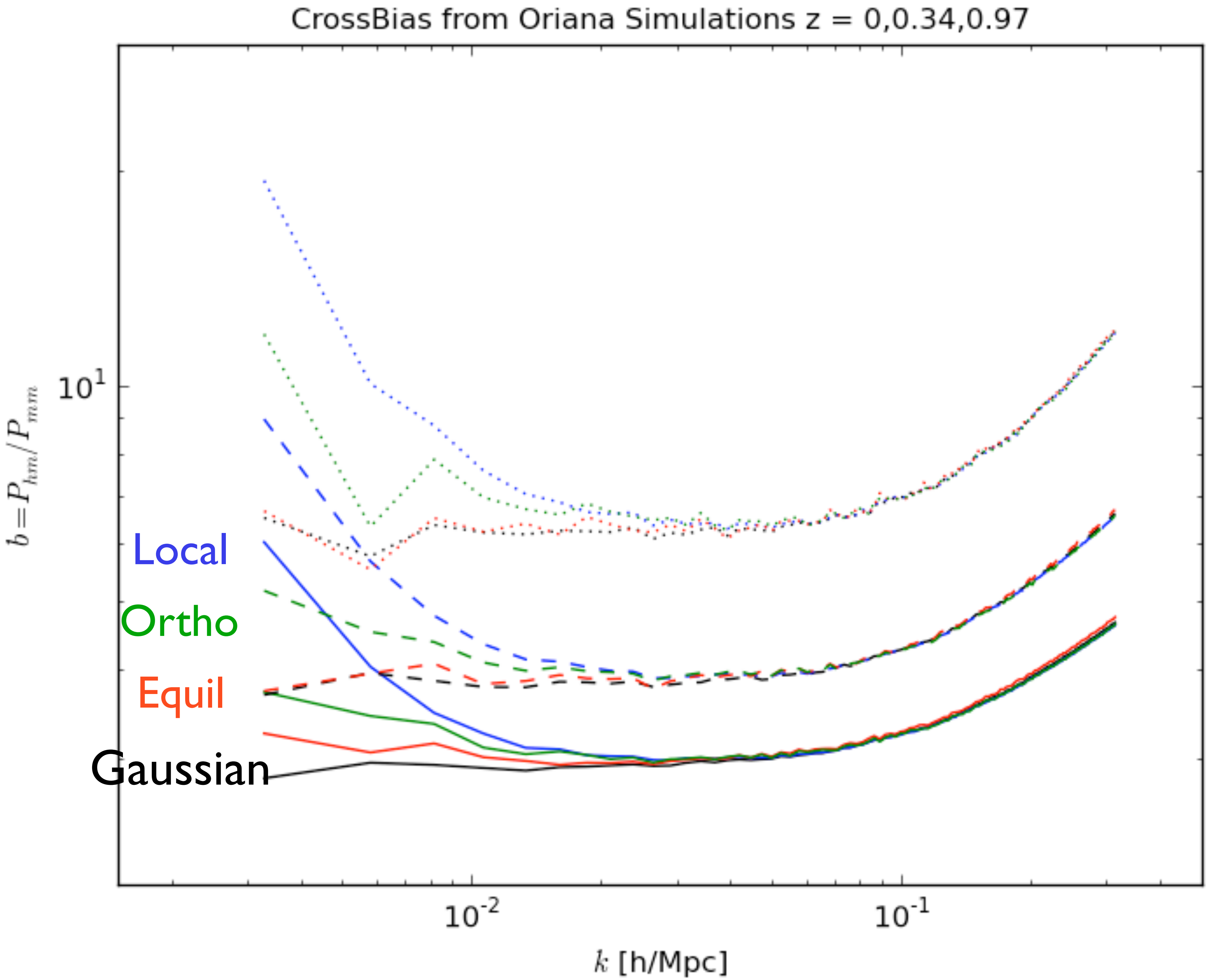

NonGaussian Matter Bispectrum (k_1=0.0628 Mpc/h k_2 = 1.5 k_1 z=0.974)
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
θ/π
$Q(k_1,k_2,\theta) - Q_{Gauss}$
Ortho
Local
Equil

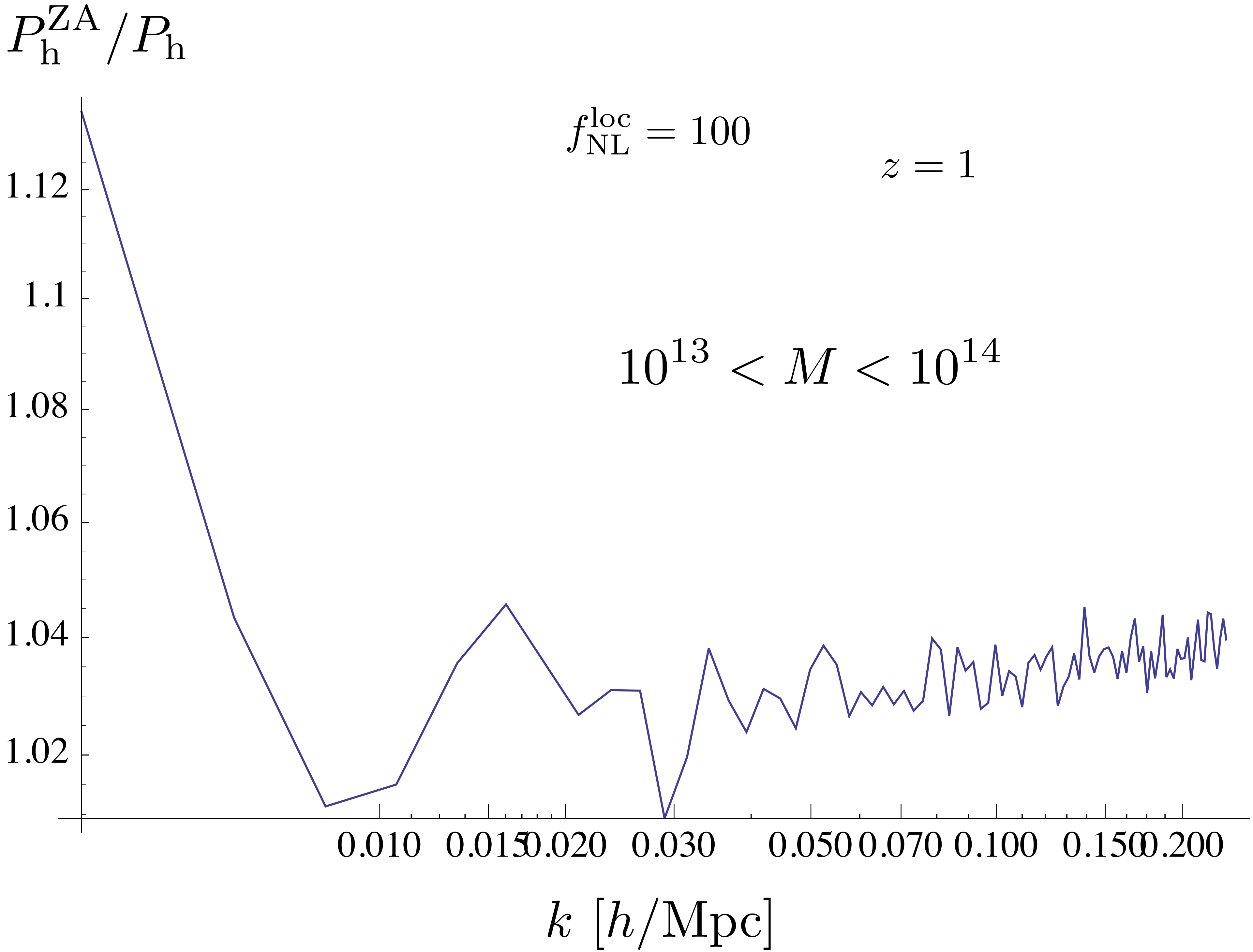


$n_{\mathrm{ZA49}}/n_{\mathrm{2LPT49}}$
1.00
0.95
0.90
0.85
0.80
0.75
$f_{\mathrm{NL}}^{\mathrm{loc}} = 100$
$z = 1$
2LPT zi=49 equivalent to:
- ZA zi=2040 (if measuring MF at z=1)
- ZA zi=4228 (if measuring MF at z=0)
1×10^{13}
2×10^{13}
5×10^{13}
1×10^{14}
2×10^{14}
5×10^{14}
1×10^{15}
$M_\odot/h$

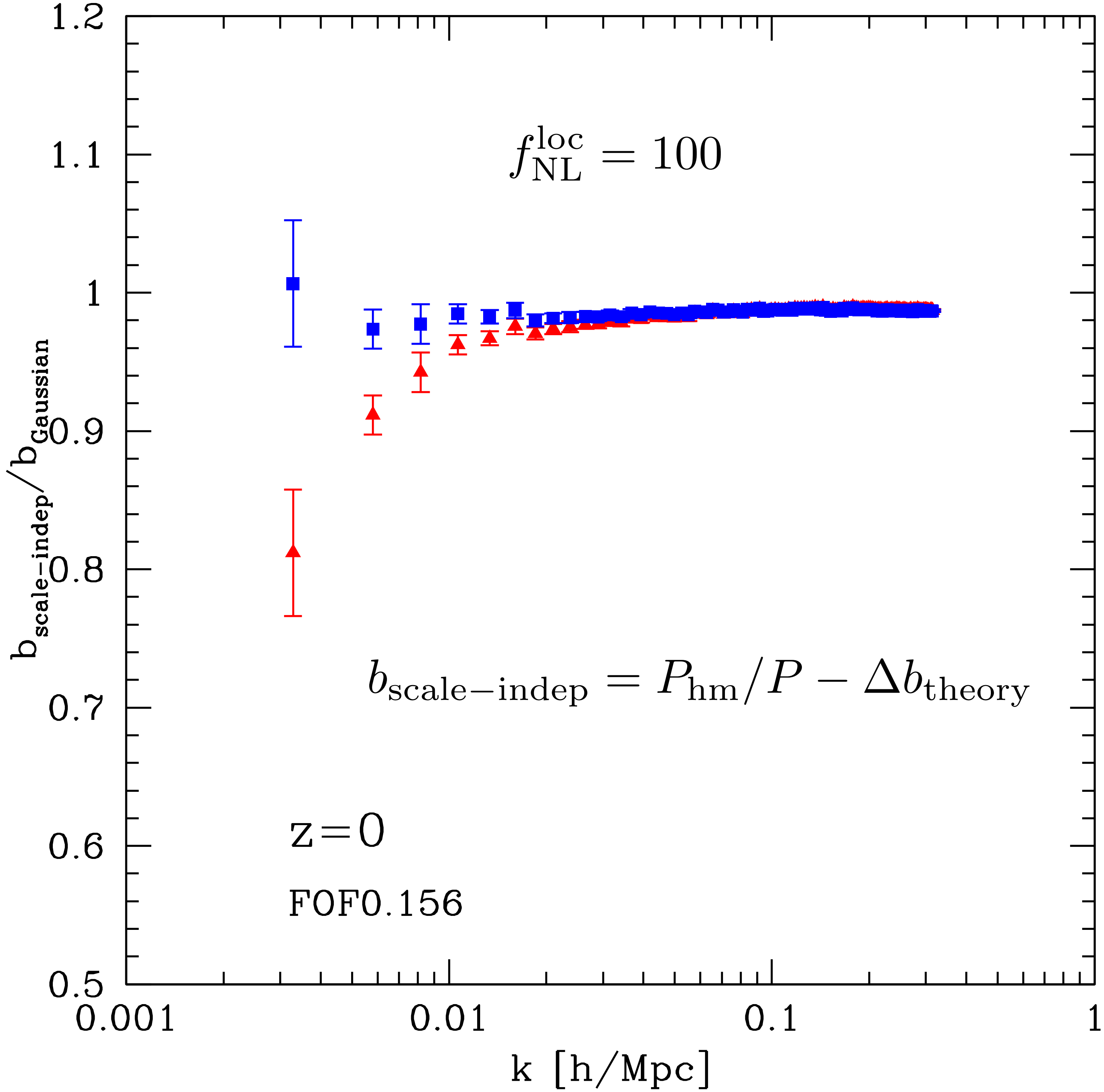


log dn/dlogM (h³Mpc⁻³)
−4
−6
−8
Warren et al.
Esmeralda
Carmen
Oriana
residual
1.4
1.3
1.2
1.1
1
0.9
13
13.5
14
14.5
15
15.5
log M (h⁻¹M⊙)


CrossBias from Oriana Simulations z = 0,0.34,0.97
$b = P_{hm}/P_{mm}$
10^1
10^{-2}
10^{-1}
k [h/Mpc]
Local
Ortho
Equil
Gaussian



$P_{\rm h}^{\rm ZA}/P_{\rm h}$
$f_{\rm NL}^{\rm loc} = 100$
$z = 1$
$10^{13} < M < 10^{14}$
1.12
1.1
1.08
1.06
1.04
1.02
0.010
0.015
0.020
0.030
0.050
0.070
0.100
0.150
0.200
$k\ [h/{\rm Mpc}]$



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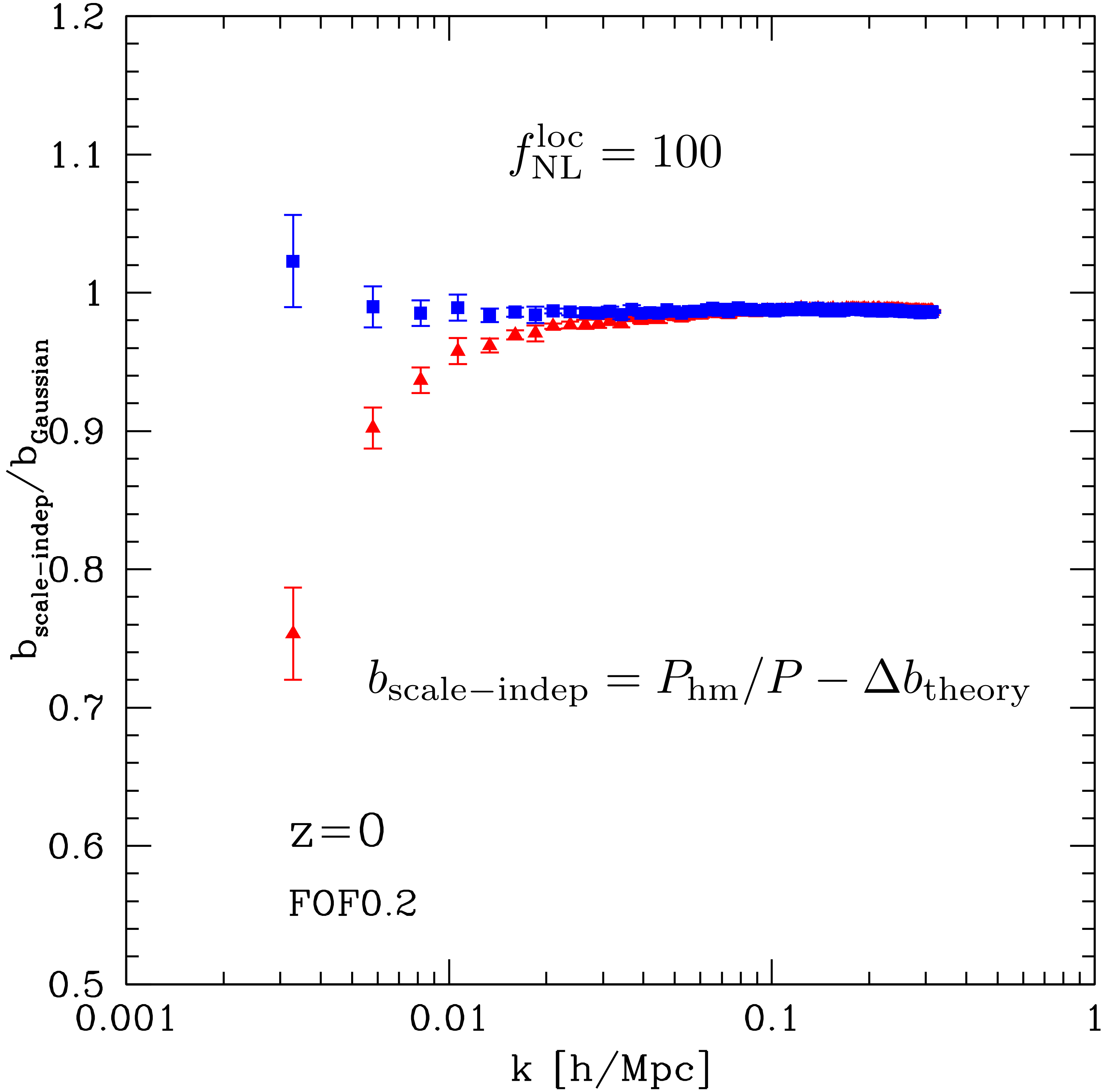
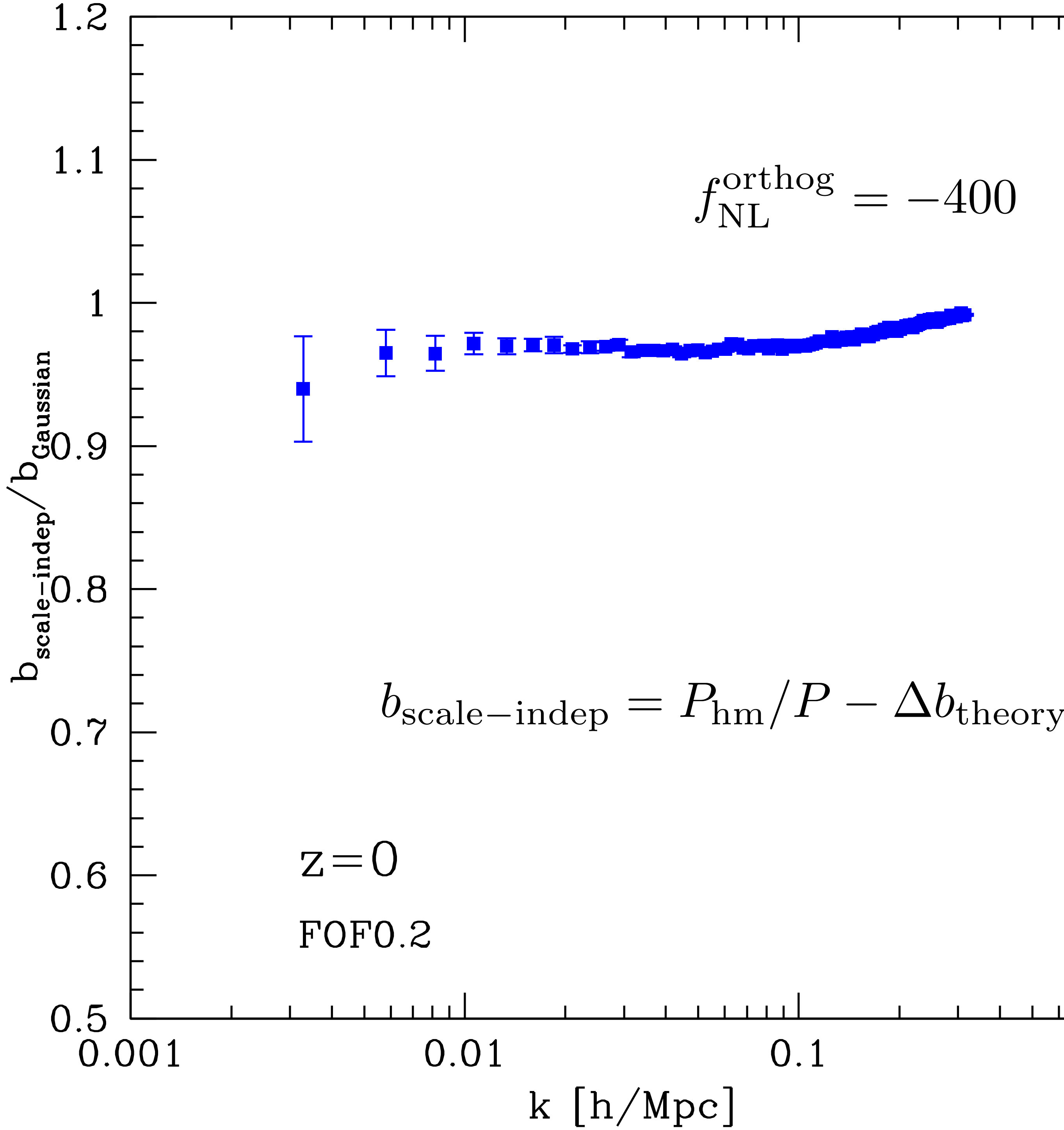


$f_{\rm NL}^{\rm loc} = 100$
$b_{\rm scale-indep} = P_{\rm hm}/P - \Delta b_{\rm theory}$
z=0
FOF0.2
$b_{\rm scale-indep}/b_{\rm Gaussian}$
k [h/Mpc]



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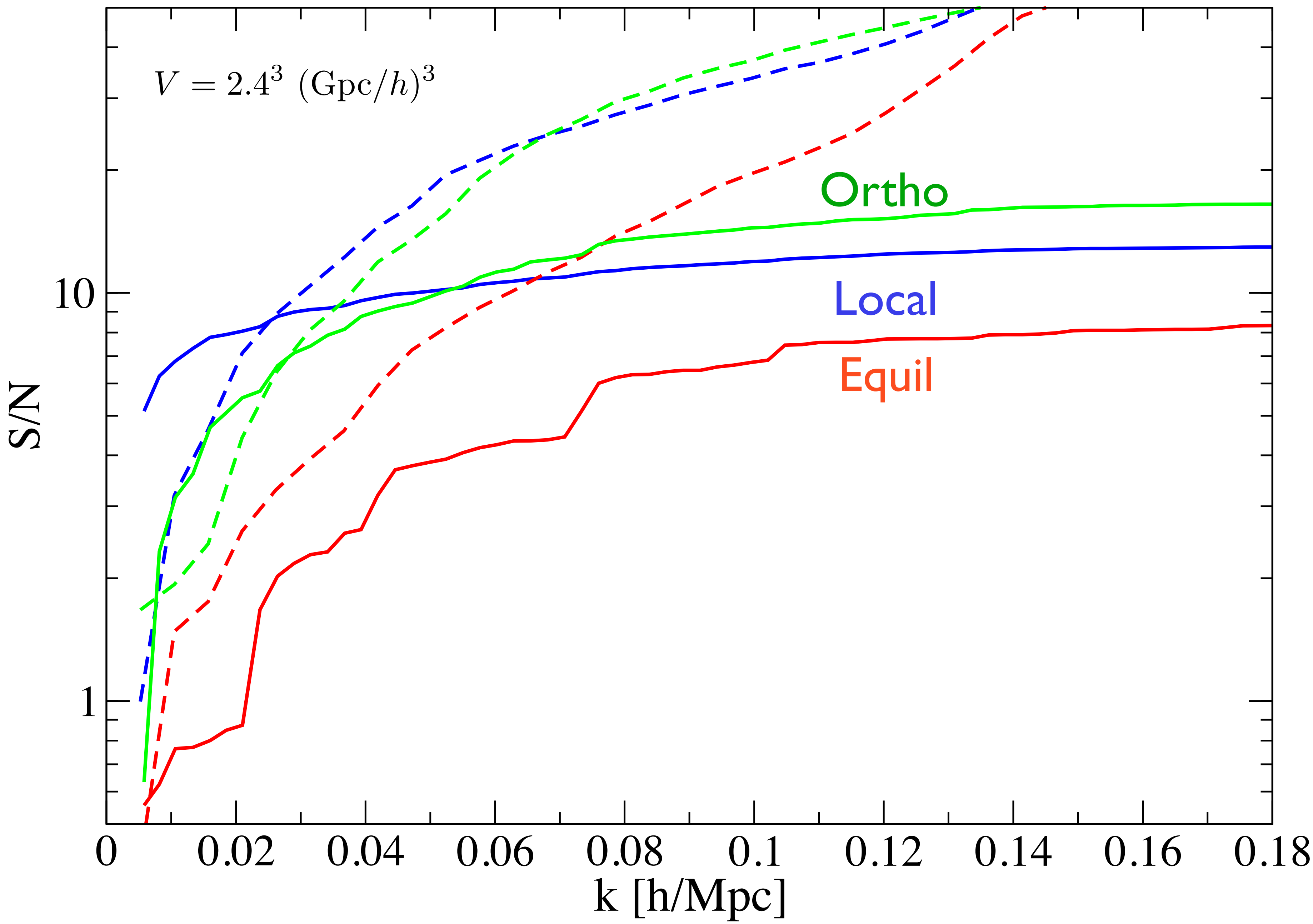
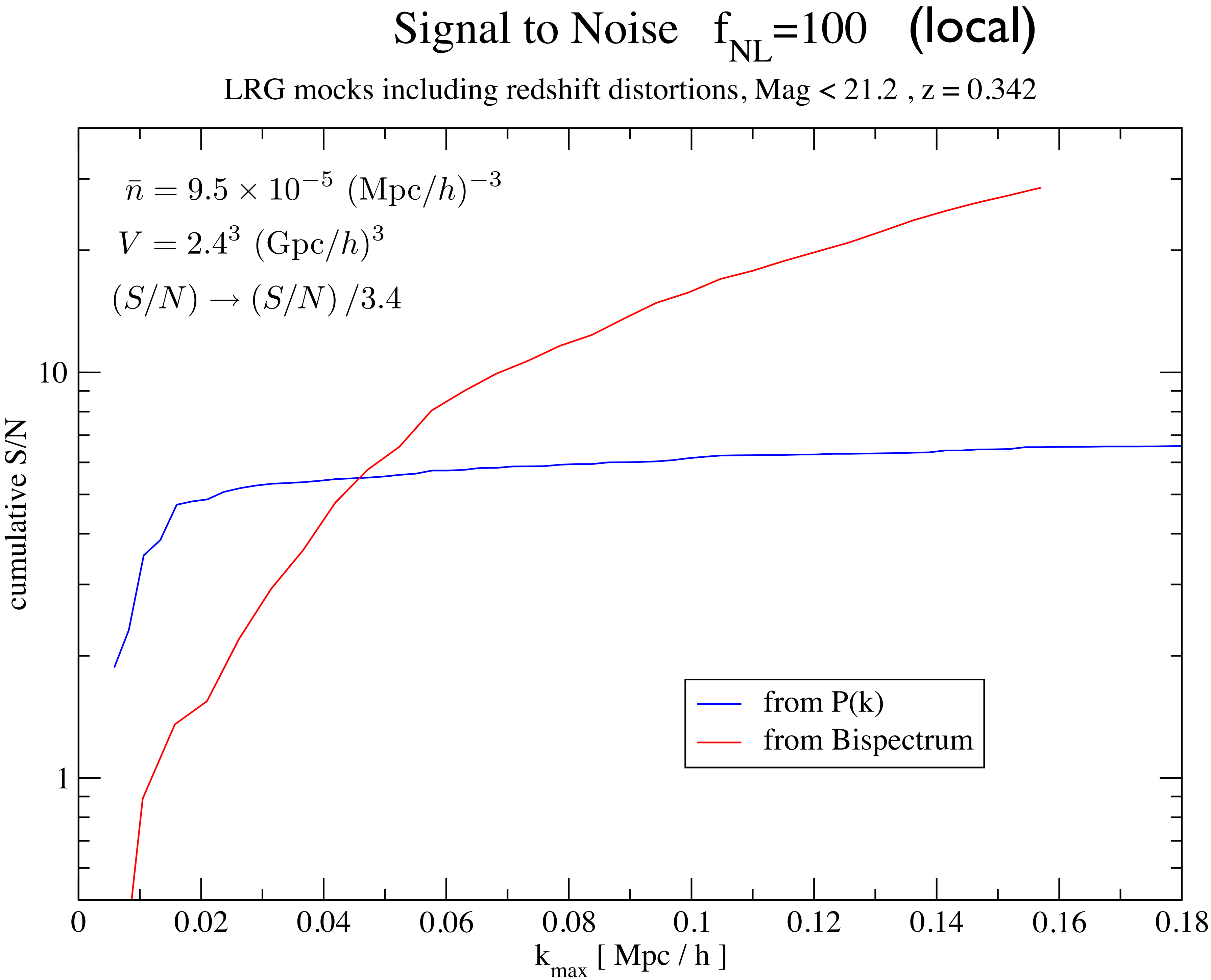
Halo S/N for Non-Gaussian Models, z=1.0, M>10^{14}Mo

dashed= from bispectrum, solid=from power



Adding Bispectrum information helps a lot...



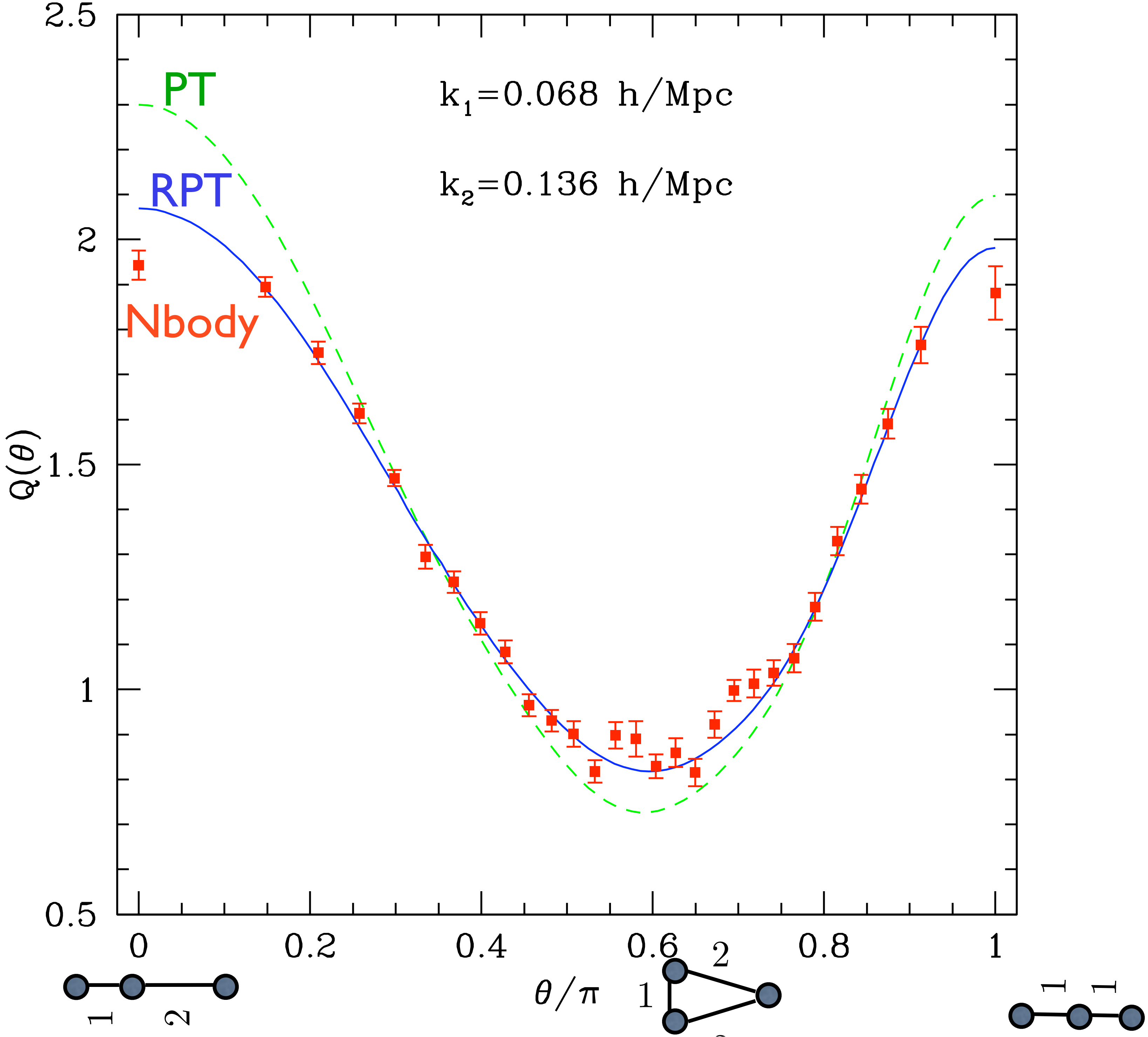
Things to worry about when using Bisp for PNG...

Shapes induced by:

- gravitational instability (2-loop RPT for the mass)
- Redshift distortions beyond PT (extension of RS 04)
- Galaxy bias beyond local approximation (even for Gaussian ICs)

- characterize Bispectrum Eigenmodes + non-Gaussian likelihood (RS 00; Gaztanaga & RS 05)

Bispectrum



PT
RPT
Nbody
k1=0.068 h/Mpc
k2=0.136 h/Mpc
Q(θ)
θ/π
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1